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Deconstructing 5D supersymmetric $U(1)$ gauge theories on orbifolds

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Abstract

We investigate deconstruction of five dimensional supersymmetric abelian gauge theories compactified on $S_1/\mathbb{Z}_2$, with various sets of bulk and matter multiplets. The problem of anomalies, chirality and stability in the deconstructed theories is discussed. We find that for most of the 5d brane/bulk matter assignments there exists the deconstructed version. There are, however, some exceptions.

Higher dimensional gauge theories offer interesting new tools to understand the roots of the Standard Model. Among other things, compactification on orbifolds is a very efficient mechanism of reducing symmetries. Moreover compactification on orbifolds is a simple mechanism to generate chirality in four dimensions. Another important virtue of higher dimensional theories is the possibility of localizing wave functions in extra dimensions. This can explain the hierarchy of various physical parameters, e.g. fermion masses, as a result of a small overlap of wave functions localized at different positions in extra dimensions. However, gauge theories in more than four dimensions are non-renormalizable and some quantum problems cannot be addressed in an unambiguous way.

It has recently been demonstrated [1, 2] that the physics of higher dimensional gauge theories can be reproduced in certain four dimensional theories with enlarged gauge symmetry. For example, the correspondence exists between five dimensional gauge theories with the gauge group G and four dimensional gauge theories with the gauge group G replicated N times, $G \times G \times \cdots \times G$. The four dimensional theory is referred to as 'latticized' or 'deconstructed' and can be viewed as a renormalizable completion of the latter. A more general view on deconstruction is that, inspired by higher dimensional gauge theories, one arrives at a class of purely 4d renormalizable gauge theories that offer (and often generalize) similar benefits to those of higher dimensional gauge theories.

Recently, some attention has been focused on 5d supersymmetric $U(1)$ gauge theories compactified on the orbifold $S_1/\mathbb{Z}_2$ (or more generally on the question of anomalies, localization and stability which is most easily studied in the $U(1)$ case). In this letter we investigate how similar properties appear in deconstruction. As a by-product of this discussion we clarify some aspects of the correspondence between the geometrical space in 5d and the 'group product space' in 4d.

We begin with recalling the construction of 5d supersymmetric $U(1)$ gauge theories on orbifolds. The 5d Abelian gauge theory in a *flat* background in the $\mathcal{N} = 1$ superspace formalism [3, 4, 5, 6] reads:

$$S_{5g} = \int \mathrm{d}^4x\mathrm{d}y \left\{ \left[\frac{1}{2}W^\alpha W_\alpha + \text{h.c.}\right]_F + \left[\left(\partial_5 V - \frac{1}{\sqrt{2}}\left(\Phi + \Phi^\dagger\right)\right)^2\right]_D \right\} . \tag{1}$$

In the above, $V = (A_\mu, \chi, D)$ is the $\mathcal{N} = 1$ vector multiplet and $\Phi = (\frac{1}{\sqrt{2}}\Sigma + i\frac{1}{\sqrt{2}}A_5, \Psi, G)$ is a chiral multiplet, singlet under $U(1)$, which completes the vector multiplet to the 5d $\mathcal{N} = 2$ multiplet.

The action for a bulk matter multiplet (called hypermultiplet) charged under $U(1)$ is given by:

$$S_{5h} = \int \mathrm{d}^4x\mathrm{d}y \left[H^\dagger e^{2g_5 qV} H + \tilde{H}^\dagger e^{-2g_5 qV}\tilde{H}\right]_D + \int \mathrm{d}^4x\mathrm{d}y \left[\sqrt{2}g_5\tilde{H}\Phi H + \tilde{H}\partial_5 H + \text{h.c.}\right]_F \tag{2}$$

where $H = (H, \psi, F)$ and $\tilde{H} = (\tilde{H}, \tilde{\psi}, \tilde{F})$ are two chiral multiplets in fundamental and anti-fundamental representation of the gauge group that make up one 5d hypermultiplet.

The pure 5d supersymmetric $U(1)$ gauge theory on $S_1/\mathbb{Z}_2$ is non-anomalous. This is because all fields of the gauge multiplet are $U(1)$ singlets. But in models with a charged hypermultiplet the orbifold projection leaves only one chiral zero-mode and so the 4d effective theory is anomalous. This anomaly manifests itself in a peculiar way in the full 5d set-up, namely, *half* of the anomaly is localized at each fixed point [7]:

$$\partial_\alpha J^\alpha = \frac{1}{2}\left[\delta(y) + \delta(y - \pi R)\right] \mathcal{Q} \tag{3}$$

where $\mathcal{Q}$ is the standard anomaly of the 4d effective theory (analogous anomalies for more complicated orbifolds are discussed in ref. [8]). This anomaly can of course be cancelled by adding another hypermultiplet with the zero mode of opposite charge. Another option is to add a chiral multiplet of opposite charge at one of the fixed points which also contribute to localized anomalies. Both possibilities lead to non-anomalous zero-mode spectrum but in the latter case, due to the factor 1/2 in eq. (3), the 5d current still looks anomalous, $\partial_\alpha J^\alpha = \frac{1}{2}[-\delta(y) + \delta(y - \pi R)]\mathcal{Q}$. However, this would-be anomaly can be removed by adding a local Chern-Simons counterterm [9, 10, 11] and does not lead to any inconsistencies of the theory. Hence, to have a non-anomalous 5d model of this type it is enough to insist on non-anomalous spectrum of the zero-modes.

It is well-known that 4d supersymmetric theories with $U(1)$ gauge symmetry allow for the presence of the $\xi[V]_D$ term in the action. In 5d models the situation is different as the symmetries ($\mathcal{N} = 2$ in the bulk and $\mathcal{N} = 1$ on the boundaries) allow only for FI terms localized at the boundaries [12], $\int \mathrm{d}^4x\mathrm{d}yD[\xi_0\delta(y) + \xi_\pi\delta(y - \pi R)]$. If we insist that supersymmetry is not spontaneously broken, the vacuum configuration must satisfy the D-flatness condition:

$$\int \mathrm{d}y \left[\xi_0\delta(y) + \xi_\pi\delta(y - \pi R) + g_5 q(H^2 - \tilde{H}^2)\right] = 0\,. \tag{4}$$

If the gauge symmetry is to stay unbroken, the hypermultiplet scalars cannot receive any vevs. Then, from the D-flatness condition it follows that the FI terms satisfy:

$$\xi_0 + \xi_\pi = 0\,. \tag{5}$$

The condition (5) translates into vanishing of an FI term in the 4d effective theory. The effect of such FI term in the 5d picture is to induce an expectation value of the gauge multiplet scalar Σ according to the equation:

$$\langle\Sigma\rangle = \frac{1}{2}\xi_\pi\epsilon(y)\,. \tag{6}$$

If only the gauge multiplet were present, the vev of Σ would have no effect whatsoever on the low-energy effective theory. In the presence of a bulk hypermultiplet, the vev of Σ induces the hypermultiplet *kink-mass* term:

$$W = \sqrt{2}g_5\Phi H\tilde{H} \to M\epsilon(y)H\tilde{H} \tag{7}$$

with $M = \frac{1}{2} g_5 \xi_\pi$. Such a kink-mass leaves the zero-mode massless while it shifts the tower of the massive KK modes, $m_n^2 = M^2 + (n/R)^2$ for $n > 0$. It also disturbs the profiles of the wave functions, in particular, it leads to an exponential localization of the zero-mode:

$$H^{(0)} = \sqrt{\frac{M}{2\left(e^{M\pi R} - 1\right)}}\, e^{M|y|}\,. \tag{8}$$

Depending on the sign of M, the zero-mode is localized either on the $y = 0$ or $y = \pi R$ brane.

It was found in ref. [9, 13] that in 5d localized FI terms can be generated dynamically. More precisely a bulk hypermultiplet with the zero-mode of charge q generates the operator

$$D g \frac{q}{2} \left\{ \frac{\Lambda^2}{16\pi^2} \left[\delta(y) + \delta(y - \pi R)\right] + \frac{\ln \Lambda^2}{64\pi^2} \left[\delta''(y) + \delta''(y - \pi R)\right] + \dots \right\}. \tag{9}$$

It has contributions localized at the orbifold fixed points which are quadratically and logarithmically sensitive to the cut off scale Λ. On the other hand a brane chiral multiplet of charge q_0, located at $y = 0$, generates just the standard FI term $D g q_0 \frac{\Lambda^2}{16\pi^2} \delta(y)$ localized at $y = 0$. This raises the question about stability of various configurations of matter fields in the 5d set-up. As we discussed previously we should concentrate on those configurations for which the sum of charges $\mathrm{Tr}\, q$ of the massless modes vanishes. Thus we can consider the following examples of just two massless fields with opposite charges:

- *Two bulk hypermultiplets with the zero-modes of opposite charges.* This configuration is perfectly stable, as the operators of eq. 9 generated by the two hypermultiplets cancel.

- *Two 4d chiral multiplets of opposite charges localized at the fixed points.* If both multiplets live at the same fixed point, the generated FI terms of course cancel. If they live at different fixed points then localized FI terms satisfying the condition (5) are generated.

- *One bulk hypermultiplet of charge q together with one brane chiral multiplet of charge $-q$ localized at $y = 0$.* In such a case the operator

$$D g \frac{q}{2} \left\{ \frac{\Lambda^2}{16\pi^2} \left[-\delta(y) + \delta(y - \pi R)\right] + \frac{\ln \Lambda^2}{64\pi^2} \left[\delta''(y) + \delta''(y - \pi R)\right] \right\} \tag{10}$$

  is generated. As a result, the profile of the hypermultiplet zero-mode is modified so that for large Λ it is sharply localized at $y = 0$ where the chiral multiplet lives. This spontaneous localization is partly due to the kink-mass for the hypermultiplet generated by the terms in (10) proportional to Λ^2 (such terms alone would lead to an exponential profile). On top of it, the δ'' terms lead to further localization of the zero-mode in a region around $y = 0$ with the thickness given by that of the fixed point brane. In addition to the localization of the zero mode, the massive Kaluza–Klein modes become very heavy with masses above the cut off scale. In such a way a bulk field effectively becomes a brane field: it is localized at a brane and it has no massive modes (below the cut off).

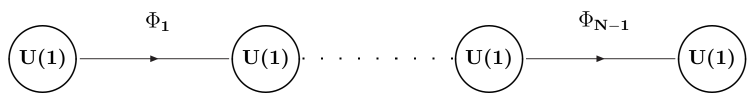

Figure 1: The quiver diagram of the model

The above simple example can be generalized to more complicated situations with more than two multiplets. In general there can be three types of fields: brane chiral multiplets at $y = 0$ with the $U(1)$ charges q_0; brane chiral multiplets at $y = \pi R$ with charges q_π; bulk hypermultiplets with zero mode charges q_B. Such a model is anomaly free and can have unbroken supersymmetry if

$$\sum q_0 + \sum q_\pi + \sum q_B = 0 \,. \tag{11}$$

Not all such models are stable. In some cases the bulk fields get localized and effectively change to brane fields. It has been shown in ref. [13] that the bulk fields are stable if the zero-mode charges sum up to zero not only globally but also locally:

$$\sum q_0 = 0 \,, \qquad \sum q_\pi = 0 \,, \qquad \sum q_B = 0 \,. \tag{12}$$

Only two of the above equalities are independent if we take into account the anomaly cancellation condition (11).

Later we will compare 5d models with models obtained in deconstruction. Such comparison should be performed at the level of the effective 4d models. Thus, one needs a criterion to distinguish the bulk fields from the brane fields from the 4d point of view. The existence of massive KK modes is such a criterion: a brane field has only the zero mode while a bulk field has a zero mode and a tower of massive modes (with masses below the cut-off or the deconstruction scale).

In the remainder of this letter we discuss the issue of FI terms, anomalies and localization in the deconstruction set-up. It has been suggested in ref. [2] (and in [14] for a supersymmetric case) that the physics of gauge theories on orbifolds can be realized in deconstruction if the quiver diagram of the 4d model is of the 'aliphatic' type, see Fig. 1. More precisely, deconstruction of 5d supersymmetric $U(1)$ gauge theory involves N $U(1)$ gauge multiplets V_p and $N-1$ chiral multiplets Φ_p (called link-Higgs) charged as $(Q, -Q)$ under the p-th and $p+1$-th gauge group, respectively. Note that such choice of the charges introduces 'orientation' in the group product space. The vacuum expectation values of the link-Higgs bosons break the product group down to the diagonal subgroup and it is below the scale set by these vevs where the correspondence holds.

The correspondence to gauge theories in a flat background is realized by assuming universal values of the gauge coupling and link-Higgs vevs, $g_p = g$, $v_p = v$ (nonuniversal values correspond to 5d gauge theories in warped backgrounds [15, 16, 17]). For deconstruction of $SU(M)$ gauge theories, arbitrary link-Higgs vevs are flat directions of the scalar potential. This is no longer the case for deconstructing $U(1)$. Note first that now FI terms for every gauge group are consistent with the symmetries, as in deconstruction we have only $\mathcal{N}=1$ supersymmetry. Adding the FI terms $\sum_p 2\xi_p[V_p]_D$ results in the scalar potential:

$$V=\frac{1}{2}g^2\left[(Q|\Phi_1|^2+\xi_1)^2+(Q|\Phi_2|^2-Q|\Phi_1|^2+\xi_2)^2+\cdots+(-Q|\Phi_{N-1}|^2+\xi_N)^2\right]. \qquad (13)$$

The first thing to see here is that if all the FI terms were set to zero, the minimum of this potential would be at $\langle\Phi_p\rangle=0$ (corresponding to an unbroken product gauge group) and there would be no energy range where the deconstruction model could match the 5d gauge theory. This situation is different from the 5d case, where the presence of FI terms is by no means necessary. Secondly, the minimum with unbroken supersymmetry satisfies:

$$\begin{aligned} Q\langle\Phi_1\rangle^2 &= -\xi_1\,, \\ Q\langle\Phi_2\rangle^2 &= -\xi_1-\xi_2\,, \\ &\dots \\ Q\langle\Phi_{N-1}\rangle^2 &= -\xi_1-\xi_2-\ldots-\xi_{N-1}=\xi_N\,. \end{aligned} \qquad (14)$$

As we are interested here in models with universal link-Higgs vevs we must further constrain $\xi_2=\ldots=\xi_{N-1}=0$, thus we must forbid the appearance of FI terms in all except the boundary gauge groups. In such a case the existence of a supersymmetric minimum requires the (fine-tuning) condition on the FI terms and additional conditions on their signs:

$$\begin{aligned} \xi_1+\xi_N &= 0\,, \\ Q\xi_1 &< 0\,. \end{aligned} \qquad (15)$$

(From the above it follows that $Q\xi_N>0$.) The former condition is clearly the analog of eq. (5) which ensures that the FI term in the effective low-energy theory vanishes. The latter has no corresponding condition in the 5d theory, which again signals that the role of FI terms in deconstruction cannot be exactly mapped on the 5d theory. At this stage the FI terms are introduced 'by hand'. Once their magnitude is chosen, the deconstruction scale is unambiguously determined, thus the arbitrariness in choosing the FI terms translates into arbitrariness of choosing the cut-off scale of the 5d theory. Observe that the FI terms generated by the link fields (which give nonzero net charges for the first and the last groups) can not be used to break the product gauge group because they do not satisfy the second of the conditions (15).

In deconstruction, all the link fields are singlets under the diagonal subgroup. But in the unbroken phase the links are charged under $U(1)$'s of the product group and, in particular, only the first ($(N-1)$–th) link-Higgs chiral multiplet transforms under the first (N–th) gauge group of the product. Thus $U(1)_1$ and $U(1)_N$ have equal in magnitude and opposite in sign anomalies. Recall that in 5d anomalies, which globally sum up to zero, can be cancelled by the addition of the local operator - the Chern-Simons term. Similarly in deconstruction we are able to cancel such anomaly without modifying the low energy spectrum. One possibility [14] is to add two chiral multiplets: S with charge $-Q$ under $U(1)_1$ and $\tilde{S}$ with charge $+Q$ under $U(1)_N$. They would emerge in the deconstruction phase as massless multiplets charged under the diagonal group and obviously have no counterparts among the 5d degrees of freedom. In order to meet the requirement that the 5d and the deconstruction model are analogous below the deconstruction scale one has to add a non-renormalizable term to the superpotential $W = \frac{1}{v^{N-2}} S\Phi_1 \ldots \Phi_{N-1}\tilde{S}$ which gives the mass of order v to S and $\tilde{S}$ multiplets. Alternatively, one can cancel anomalies via Wess-Zumino terms. It was shown that one can choose a WZ term such that its gauge variation is (for notation and details see refs. [18, 19, 20])

$$\delta\mathcal{L}_{WZ} = -\frac{1}{24\pi^2}(\epsilon_p dA_p dA_p - \epsilon_{p+1} dA_{p+1} dA_{p+1})\,. \tag{16}$$

Thus those anomalies which sum up to zero in the diagonal subgroup can be cancelled. However, cancelling the anomalies via WZ terms leaves non-zero Trq in the boundary groups, which destabilizes the FI terms and, in consequence, the deconstruction scale. Therefore we always assume that anomalies in the deconstructed pure gauge theory are cancelled by the $(S, \tilde{S})$ pair.

Now we turn our attention to the physics of a 5d hypermultiplet realized in deconstruction. In order to mimic hypermultiplets one introduces [14] two sets of chiral multiplets, $H_p = (H_p, \psi_p)$ and $\tilde{H}_p = (\tilde{H}_p, \tilde{\psi}_p)$ (later called 'replicated multiplets'), with charge Q and $-Q$ with respect to the p-th gauge group. The most general renormalizable superpotential with mass terms and couplings independent of p is the following

$$W = \sum_{p=1}^{N-1} \sqrt{2}\lambda \tilde{H}_p \Phi_p H_{p+1} - \sum_{p=1}^{N} m \tilde{H}_p H_p\,. \tag{17}$$

The Yukawa coupling λ should be fine-tuned to the gauge coupling, $\lambda = g$, in order to match the $\mathcal{N} = 2$ supersymmetric interactions of the 5d theory. Furthermore, the fine-tuning of the mass parameter m to the link-Higgs vev, $m = gv$, leads to the similar spectrum and interactions as those of a 5d bulk hypermultiplet without a kink-mass term. Note also that, with such set of links, we can only reproduce hypermultiplets with charge Q. If we allow for non-renormalizable interactions in the superpotential all rational charges are allowed.

When deconstructing orbifold theories, one has to set either $\tilde{H}_N \equiv 0$ or $H_1 \equiv 0$. The first choice results in the zero mode of charge Q under the diagonal group while the second yields the zero mode of charge $-Q$. This way one introduces chirality in the matter sector. In 5d gauge theories chirality appears due to the $\mathbb{Z}_2$ symmetry (or boundary conditions in the 'downstairs picture') which removes some of the degrees of freedom from the spectrum. $\mathbb{Z}_2$ acts differently on left- and right-handed fermion components and, in particular, it leaves in the spectrum only one chiral component of the zero mode. In deconstruction we have neither $\mathbb{Z}_2$ nor boundaries to define the boundary conditions and chirality must be introduced 'by hand'. For the case without matter multiplets this step is fairly straightforward. Going from the periodic to the aliphatic quiver diagram consists in removing one chiral multiplet Φ_N which results in an odd number of chiral fermions in the theory (N gauginos and $N-1$ link Higgsinos). Thus the intuitive step of turning the 'circle' into a 'line' automatically introduces chirality as well. When the replicated matter fields H_p, $\tilde{H}_p$ are present one can apparently remove many different chiral multiplets to introduce chirality. However these possibilities are not equivalent and *only* removing $\tilde{H}_N$ or H_1 yields, in the deconstruction phase, the spectrum and interactions similar to those of the 5d case. For example, the other intuitive possibilities - removing $\tilde{H}_1$ or H_N - yield the low-energy spectrum which does not correspond to any 5d model. The difference can be seen already at the level of the superpotential (17). By removing $\tilde{H}_N$ (or H_1) we remove just one mass term from the superpotential while by removing H_N (or $\tilde{H}_1$) we remove one mass term and also one Yukawa coupling term.

Thus the two 'boundaries' of the group product space are by no means equivalent. This is obviously counterintuitive to any geometric interpretation of the group product space. Moreover it once again illustrates the fact that, in case of orbifold theories, the correspondence between 5d and deconstruction holds at the level of the effective low-energy theory only for some specific choices made during construction of the model.

Removing the anti-fundamental chiral multiplet $\tilde{H}_N$ leads to an anomaly localized at the N-th site of the group product space, similarly, removing H_1 yields an anomaly of $U(1)_1$. Below the deconstruction scale the anomaly of the diagonal group *globally* matches the anomaly of the effective model obtained from the 5d theory with one hypermultiplet. But above that scale the situation looks different than in 5d, where half of the anomaly is localized at each fixed point. This difference originates from the fact that in the deconstruction set-up the group product space is oriented and gives an oriented line in the aliphatic case while both fixed points of the orbifold are equivalent.

It is possible to calculate the spectrum and the mode decomposition of the matter fields H and $\tilde{H}$. The case with $m = gv$ was studied in [14] and the correspondence to the 5d *massless* bulk hypermultiplet spectrum was shown. We shall see that $m \neq gv$ corresponds to a bulk hypermultiplet with a kink-mass term.

First, there is one combination of the H_p multiplets which remains massless in the deconstruction phase. This is readily understood as we have an odd number of chiral matter multiplets in the theory so at least one chiral multiplet must remain massless. The zero-mode profile is

$$H^{(0)} = \sqrt{\frac{(m/gv)^2 - 1}{(m/gv)^{2N} - 1}} \sum_{p=1}^{N} \left(\frac{m}{gv}\right)^{p-1} H_p \tag{18}$$

for the zero-mode of charge $+Q$ and

$$\tilde{H}^{(0)} = \sqrt{\frac{(gv/m)^2 - 1}{(gv/m)^{2N} - 1}} \sum_{p=1}^{N} \left(\frac{gv}{m}\right)^{p-1} \tilde{H}_p \tag{19}$$

for the zero-mode of charge $-Q$. For $m < gv$ the $+Q$ ($-Q$) zero-mode is localized near the first (N-th) site, while for $m > gv$ it is localized near the N-th (first) site .

The remaining combinations of H and $\tilde{H}$ multiplets become massive. Their masses organize themselves into a tower according to the equation

$$m_n^2 = g^2v^2 \left[(1 - \frac{m}{gv})^2 + 4\frac{m}{gv} \sin^2 \left(\frac{n\pi}{2N} \right) \right] , \tag{20}$$

while the decomposition of the n-th level massive mode is (only formulae for the case of $+Q$ zero mode are given)

$$H^{(n)} = \sqrt{\frac{2}{N}} \frac{gv}{m_n} \sum_{p=1}^{N} \left[2 \sin \left(\frac{n\pi}{2N} \right) \cos \left(\frac{m\pi}{2N}(2p-1) \right) + (m - gv) \sin \left(\frac{n\pi}{N} \right) p \right] H_p \,, \tag{21}$$

$$\tilde{H}^{(n)} = \sqrt{\frac{2}{N}} \sum_{p=1}^{N-1} \left[\sin \left(\frac{n\pi}{N} \right) p \right] \tilde{H}_p \,. \tag{22}$$

For $n \ll N$ and $|m - gv| \ll gv$ the mass formula (20) becomes $m_n^2 \approx (m-gv)^2 + \frac{n^2\pi^2}{N^2} g^2v^2$ which is precisely the spectrum of a 5d hypermultiplet with the kink-mass $M = (m - gv)$ for a compactification radius $1/R = \pi gv/N$. The correspondence of the spectra holds for $M \ll gv$, that is for a kink-mass much smaller than the deconstruction scale (interpreted as the cut-off Λ of the 5d theory). For $M \sim gv$ the massive spectra of the 5d and the deconstruction models differ significantly. This could be expected, as deconstruction can reproduce the features of the 5d theory only much below Λ.

One can also easily verify that the eigenmode decomposition is analogous as in the 5d case. In this sense the correspondence between the physical space and the group product space holds for deconstruction of orbifold theories. Although the high-energy details are different in the two theories, the 5d KK mode profiles can be mapped onto mode decomposition in deconstruction and the precision of the mapping is of the order of the 'lattice spacing' $\Delta y = (gv)^{-1}$.

In 5d scenarios, apart from bulk hypermultiplets one often considers chiral multiplets localized at the $\mathbb{Z}_2$ fixed points. It is rather intuitive that the corresponding objects in deconstruction are chiral multiplets charged under the first or the N-th group. Furthermore such multiplets should not be coupled via the link-Higgs fields to multiplets living at other sites; otherwise such multiplets would be removed from the low-energy spectrum, similarly as it happened for the anomaly cancelling $(S, \tilde{S})$ pair.

The correspondence can be seen in a more formal way. Consider a multiplet P charged under the i-th $U(1)$ group only. This means it couples to the i-th gauge field, for example:

$$S = \int \mathrm{d}^4 x iqgP^\dagger \partial_\mu P A_{\mu,i} \,. \tag{23}$$

When the mode decomposition of the gauge field is inserted, this coupling becomes:

$$S = \int \mathrm{d}^4 x \sum_{n=0}^{N-1} \sqrt{\frac{2}{N}} iqgP^\dagger \partial_\mu P A_\mu^{(n)} \eta_n \cos \frac{n(2i-1)\pi}{2N} \,, \tag{24}$$

where $\eta_n = 1\sqrt{2}^{\delta n0}$. Now, recall that the analogous coupling of the chiral multiplet localized at the brane at $y = y_i$ to the KK tower of the 5d gauge field is:

$$S = \int \mathrm{d}^4 x \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} i q g_5 P^\dagger \partial_\mu P A_\mu^{(n)} \eta_n \cos\frac{n y_i}{R} \,. \tag{25}$$

We can see that in deconstruction a multiplet at the i-th site couples analogously as a brane field at the brane position $y_i = \frac{i-1/2}{gv}$. In particular, a multiplet at the first site corresponds not exactly to the boundary multiplet at $y = 0$ but to the brane multiplet at $y = \frac{1}{2gv}$. Thus in deconstruction the fixed point is resolved only up to the distance scale $\Delta y = \frac{1}{gv}$. One of the consequences of this is that it is not possible to reproduce the effects of the δ'' terms present in the 5d FI terms (9). Only exponential localization may take place in deconstruction while in 5d models also a sharp δ–like localization is possible.

We are now ready to discuss the question of stability of various configurations of 'bulk' and 'brane' matter multiplets which yield a non-anomalous spectrum of the zero modes, similarly as it was done in the 5d case. We start with the example of one replicated $(H_p, \tilde{H}_p)$ multiplet with the $+Q$ charged zero mode (the $+Q$ charged zero mode is obtained by removing $\tilde{H}_N$) and one brane multiplet P with charge $-Q$ living at the first site. In such configuration, the opposite in sign anomalies are localized at both ends of the group product space. Such globally vanishing anomaly can be cancelled via the Wess-Zumino term, but at one loop the FI term will receive corrections. These corrections will respect the relation $\xi_1 + \xi_N = 0$ and supersymmetry will stay unbroken. However, as the absolute value of the FI terms are shifted, the link-Higgs vevs are shifted too and the fine-tuning between the m parameter and gv no longer holds. In the case at hand the net charge in the N-th group is positive, thus ξ_N will increase and we obtain $m < gv$ at one loop. This result in an analogous effect as that noticed in ref. [13], namely, in spontaneous localization of the zero-mode of H at the site where P resides. Note however that the similar configuration, with P living at the N-th site is perfectly stable, and so it does not correspond to any 5d configuration.

The other examples of matter field configurations discussed in the 5d case also have their analogues in deconstruction. Two chiral multiplets living at the different fixed points translate into one chiral multiplet living at the first site and one chiral multiplet of opposite charge living at the N-th site. Similarly as in the previous case, FI terms generated at one loop will shift the link-Higgs vev and may lead to spontaneous localization of any replicated multiplets present. Less straightforward is the case with two replicated multiplets $(H_p, \tilde{H}_p)$ and $(G_p, \tilde{G}_p)$. As discussed, non-anomalous zero-mode spectrum is obtained by removing $\tilde{H}_N$ and G_1. But this step leads to non-zero, opposite in sign anomalies localized at the two endpoints of the group product space. One can cancel these anomalies via WZ terms, but again one loop corrections will destabilize the FI terms. The resulting shift of the link-Higgs vevs leads to localization of the zero-mode of H and G at the opposite endpoints. This situation is obviously different than in the 5d case.

More generally, one can formulate the condition for the stability of the bulk fields in various 'bulk-brane' configurations in deconstruction. Formulated at the level of the theory above the deconstruction scale it says that charges of all chiral multiplets have to cancell out locally in the group product space. We can also easily rephrase this condition in terms of the effective theory below the deconstruction scale. Denoting by q_1 (q_N) the charges of chiral multiplets living on the first (N-th) site and by q_+, q_- the positive and negative charges of the zero modes of the replicated multiplets, the stability conditions are

$$\sum q_- + q_1 = 0\,, \qquad\qquad \sum q_+ + q_N = 0\,. \tag{26}$$

As in the 5d case they are more restrictive than merely the condition for anomaly cancellation $\sum q_- + q_+ + q_1 + q_N = 0$. However their form is quite different than the corresponding one in 5d. Three equalities (12) must be fulfilled in 5d models to assure local cancellation of the zero-mode charges, while there are only two conditions (26) in deconstruction. The conditions in 5d relate charges of multiplets with a definite extension along the 5–th dimension (localized at one brane, localized at the second brane or propagating in the bulk). In deconstruction the positively charged bulk fields are related to some of the brane fields while the negatively charged bulk fields are related to the remaining brane fields. This difference is due to the orientation of the group product space which is determined by charges of the link fields. For some aspects of the theory the end points of the aliphatic quiver diagram can be interpreted as the end points of the orbifold but for other aspects they lose this interpretation. Then they are related rather to the two possible signs of the U(1) charges.

In summary, we have investigated deconstruction of 5–dimensional U(1) gauge theories compactified on $S^1/\mathbb{Z}_2$ with various sets of bulk and matter multiplets. We found that for most configurations of multiplets the 5d theory has its deconstructed version. There are however also some exceptions.

In both theories localized FI terms and localized anomalies (localized in the 5–th dimension or in the group product space, respectively) are generated for general sets of bulk and brane matter multiplets. If such would be anomalies sum up to zero in the effective 4d theory they can be cancelled by local counterterms without changing the low energy spectrum. In 5d theories the Chern–Simons terms can be used for that purpose. In deconstruction there are two mechanisms at our disposal. One is a direct analogue of the 5d mechanism and consists in adding Wess–Zumino terms. The second possibility is to add a pair of chiral fields with opposite charges and located at the two end points of the group product space. These two methods are not equivalent. The latter influences not only the anomalies but also the FI terms generated for the first and the last groups from the group product space.

In both types of models the necessary condition for unbroken supersymmetry is the same as the condition for absence of anomalies: the sum of charges (in deconstruction, under the diagonal group) of all zero modes must be zero.

Another similarity between the 5d and deconstructed models is that in both cases the localized FI terms lead to the localization of the bulk fields. Typically the bulk fields of one sign of the charge of the zero modes are localized around one of the branes while that with the opposite sign are localized in the vicinity of the other brane. But the form of such localization can be different in the two types of models. In deconstruction the localization is always exponential. In 5d models there are more possibilities and in some situations the localization is sharp - δ-like.

The conditions for the localization differ for the two cases. In 5d models such conditions relate zero mode charges, separately for the bulk and for each of the branes, as shown in eq. (12). In deconstruction they are given by eq. (26) and relate charges of some of the brane fields with positively charged bulk fields (and the remaining brane fields with the negatively charged bulk fields). In consequence, the 5d theory with certain sets of multiplets, e.g. with two hypermultiplets with zero-modes of opposite charges, does not have its deconstructed analogue. Our analysis shows that the geometric interpretation of the group product space can be used only for some aspects of such models.

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